Refractive Height Fields from Single and Multiple Images

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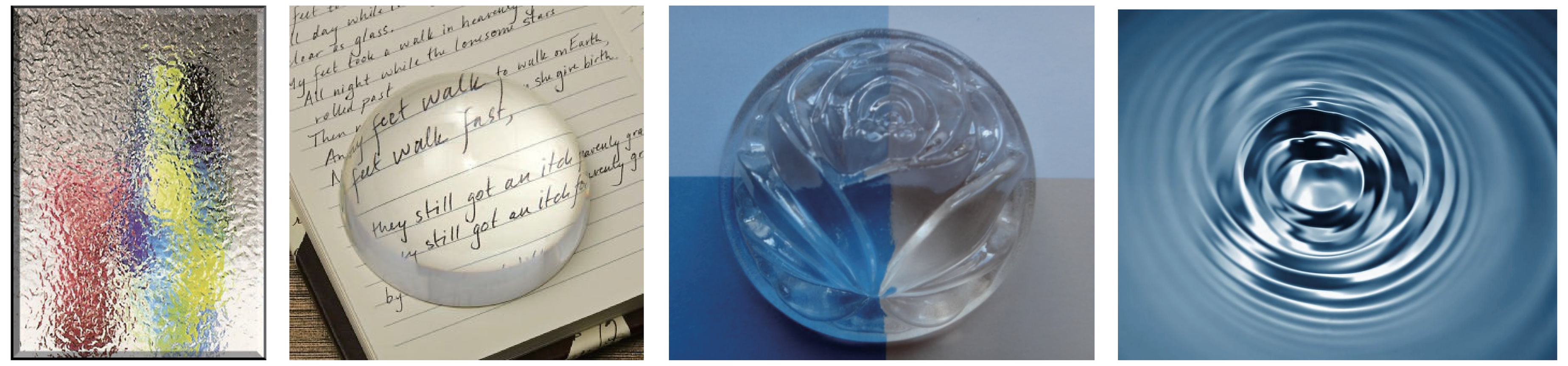
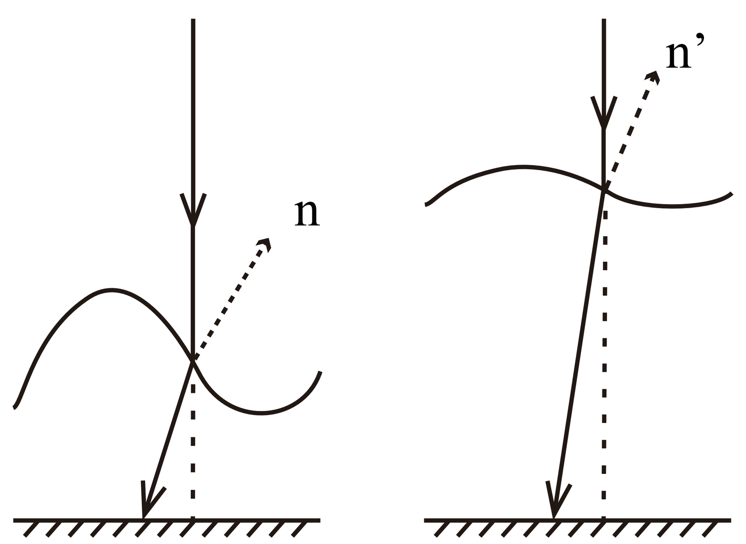
Introduction

We develop a novel framework for reconstructing homogenous, transparent, refractive height fields from a single viewpoint against known planar backgrounds.



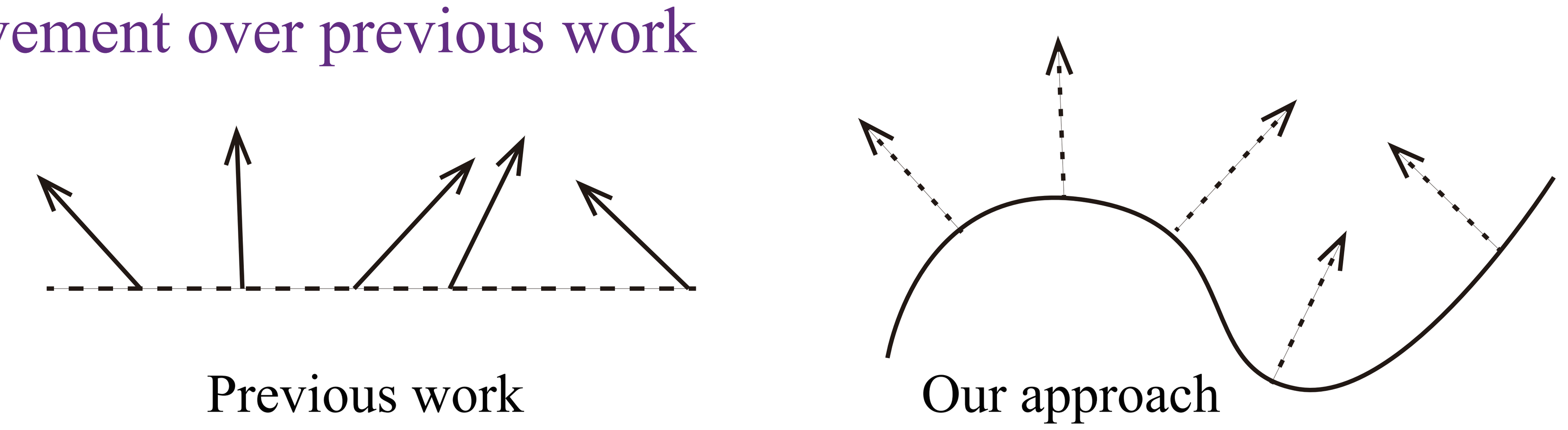
Existing approaches perform a point-by-point reconstruction which is easy to compute but known to have intractable ambiguities. Our method optimizes for the entire height field at the same time. The formulation supports shape recovery from measured distortions (deflections) or directly from the images themselves, including from a single image.

Height-normal ambiguity



Assumptions: Orthographic projection
No interreflections
Differentiable surface

Improvement over previous work



Previous work: Initially assume surface is flat with given height and index of refraction, estimate normals from deflections (how the background is distorted), then integrate normals to compute a final surface. This approach is mathematically incorrect.

Our approach: Directly optimize height and index of refraction from deflections or from images, normals are by-products.

[1] H. Murase. Surface shape reconstruction of a nonrigid transparent object using refraction and motion. TPAMI, pages 1045–1052, 1992.

Height field from a deflection map

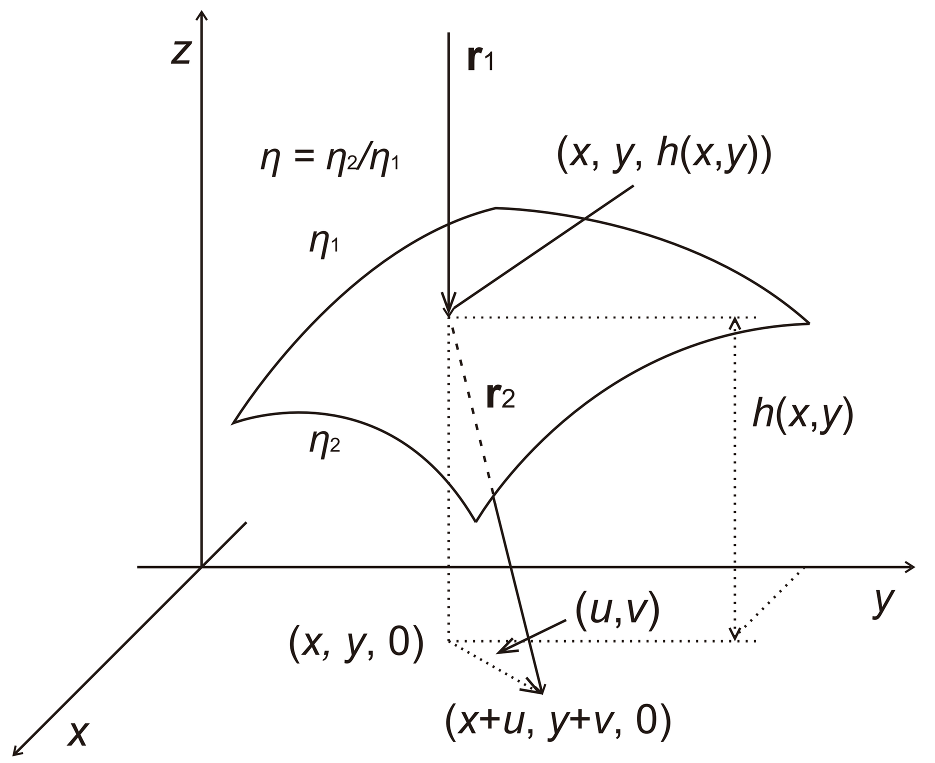
Relating deflection (u,v) to height h(x,y)

Starting from Snell's law, we can derive a relation between deflections and heights.

$$[u, v] = \gamma h [h_x, h_y]$$

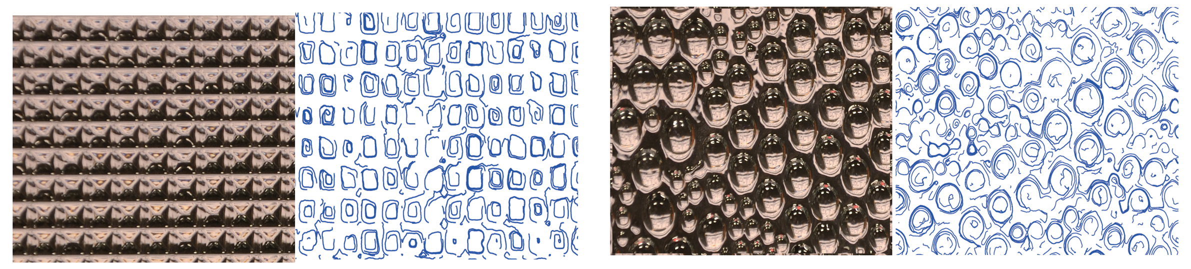
where

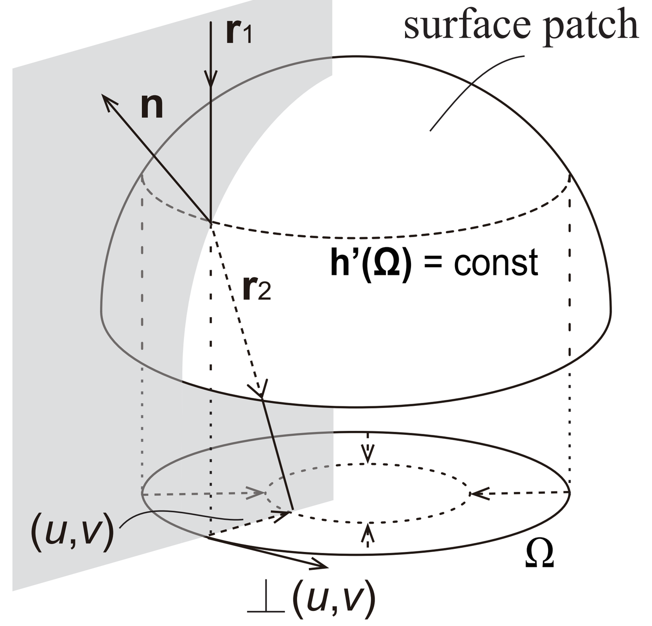
$$\gamma = \frac{-1 + \sqrt{(\eta^2 - 1)(h_x^2 + h_y^2 + 1) + 1}}{h_x^2 + h_y^2 + \sqrt{(\eta^2 - 1)(h_x^2 + h_y^2 + 1) + 1}}$$



Isocontour regularization

We derive a new regularization term based on the ***characteristic curves*** (isocontours).
n, **r**$_1$, and **r**$_2$ are co-planar. Their projections on xy-plane are collinear.
The projection of **n** is collinear with ∇h. The projection of **r**$_2$ is collinear with (u,v).
Thus (u,v) is collinear with ∇h, so (u,v) is orthogonal to the isocontour of h.





Objective:

$$E(h, \eta) = \sum_{x,y} (u - \gamma h h_x)^2 + (v - \gamma h h_y)^2 + w \sum_{x,y} \sum_{x',y' \in \Omega(x,y)} (h(x,y) - h(x',y'))^2$$

Height field from a single image

Instead of first recovering a deflection map (two variables per pixel) and then estimating the height field from it, we can write the observed image as a function of the height field, and then solve for the height field that best explains the observed image by optimizing the following objective.

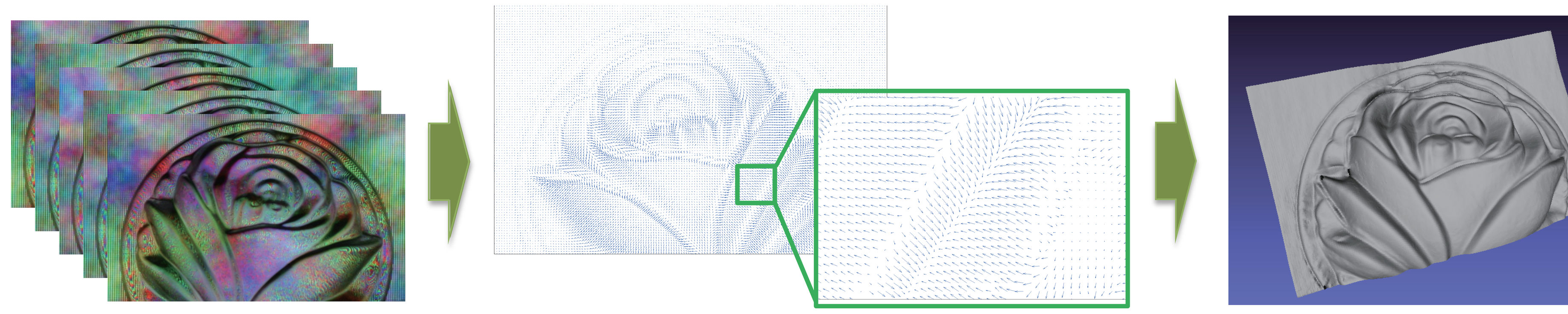
Objective:

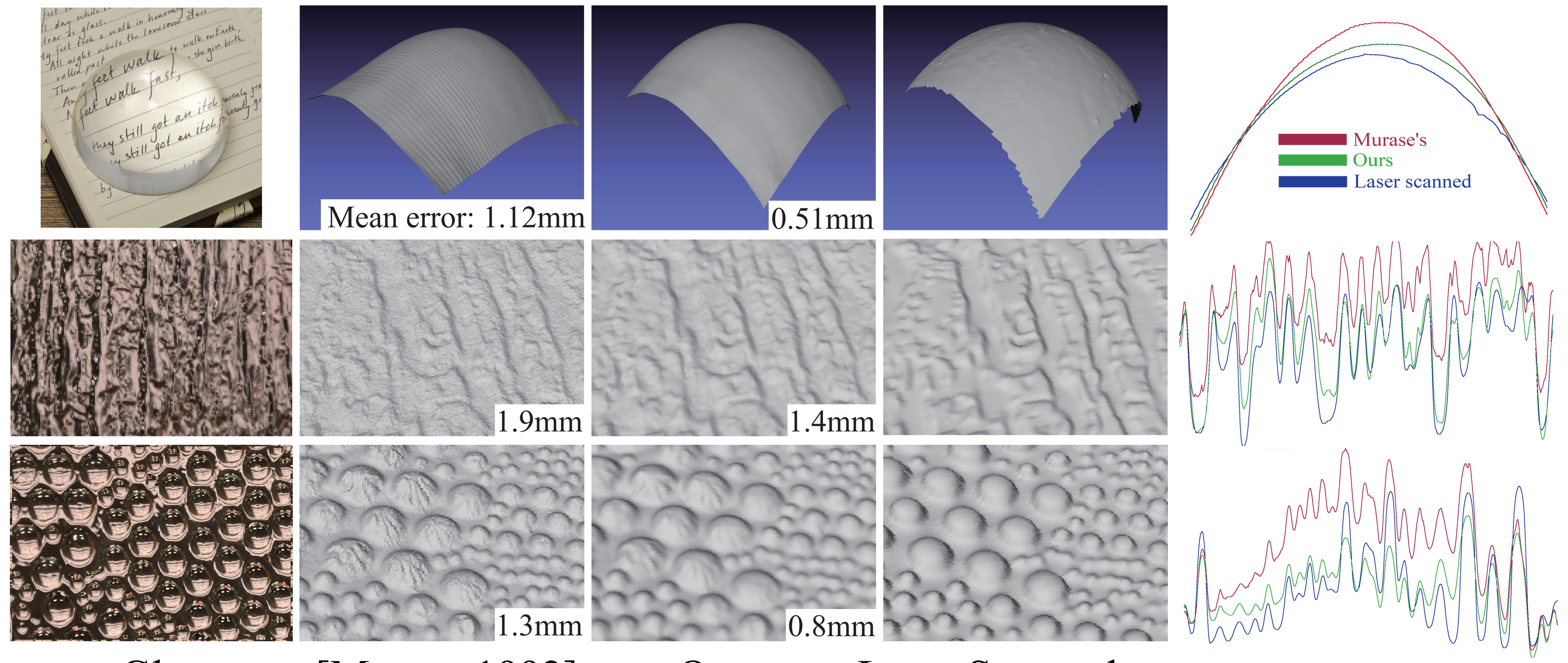
$$E(h, \eta) = \sum_{x,y} \| I(x,y) - B(x + \gamma h h_x, y + \gamma h h_y) \|_2^2$$

Optimization

We solve for h and η using a multiscale optimization and gradient descent.
Running time for a 600x400 height map:
Deflection based: ~1.5 hours Sinlge image based: ~2.5 hours

From deflections (multiple images)

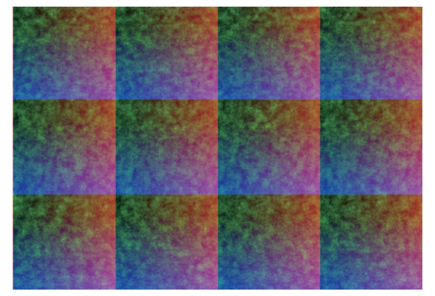


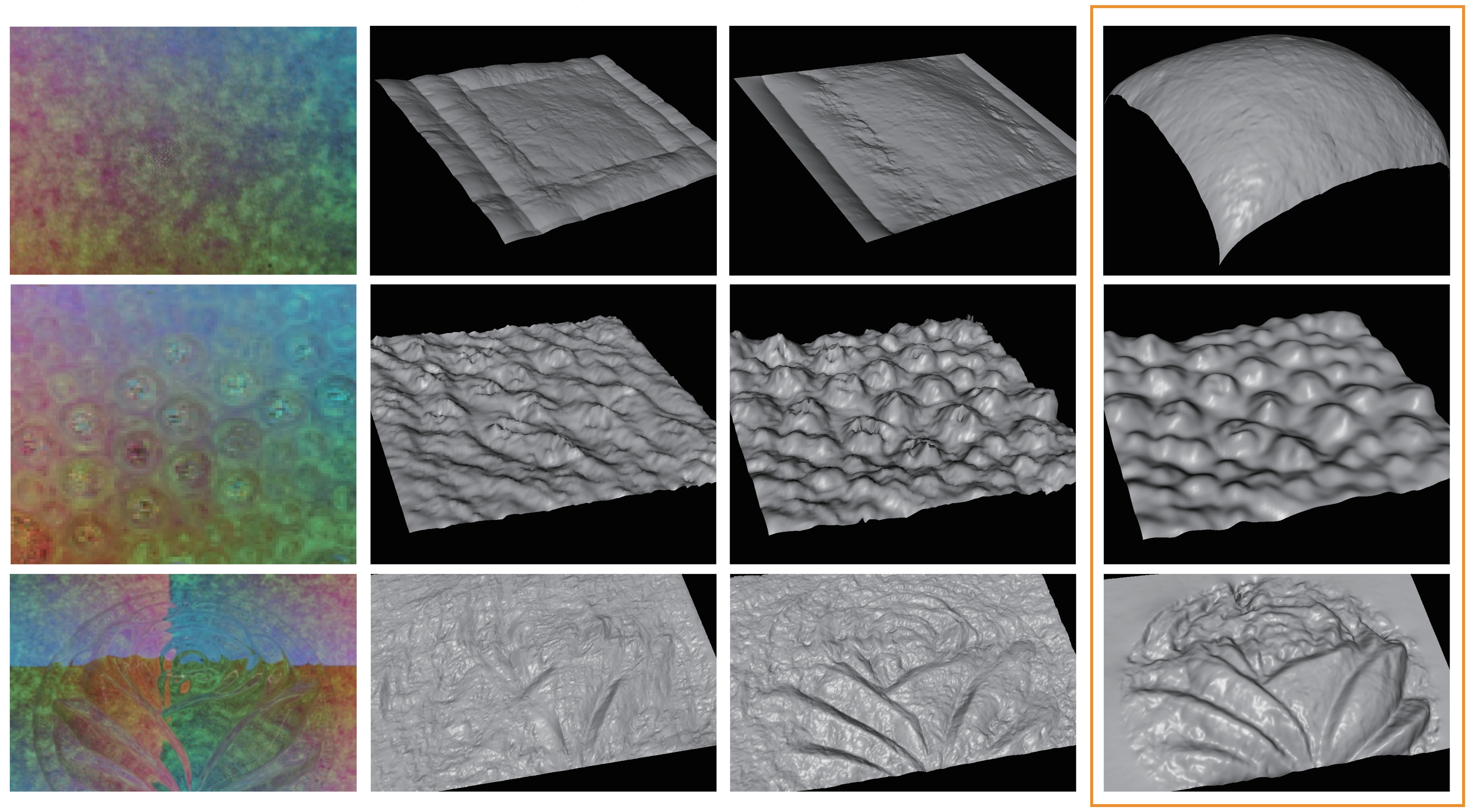


Glass [Murase 1992] Ours Laser Scanned

From single image

Background pattern: Red: horizontal coding
Blue: vertical coding
Green: high-pass filtered Perlin noise





Captured image [Murase 1992] Our deflection based Our final surfaces