Carpentry Compiler: Supplemental Material

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1 OPTIMIZATION RESULTS
In this section, we provide additional details and results of our optimization method for generating fabrication instructions for all the models in Figure 9.

1.1 Cost Metric Parameters
The material cost is measured as the sum of the costs of all used lumber pieces, \( l_i, i = 1, \ldots, N \). The price of each lumber, \( P(l_i) \), depends on its dimension and is given in Table S1. The prices are relative and are computed based on prices from standard US vendors.

\[
f_c = \sum_{i=1}^{n} P(l_i).
\]

To compute \( f_c \), we assign fabrication times to each tool based on the complexity of the setup and operating processes as reported by our carpentry experts. Chopsaw has the simplest setup process and therefore the least setup time. It is followed by bandsaw and jigsaw have a similar setup process where the path has to be precisely marked on the part. Both are more time consuming than setting up a chopsaw. Tracksaw has the most difficult setup and is thus assigned the largest time. It is followed by bandsaw and jigsaw respectively.

\[
f_t = \sum_{k=1}^{m} n_k \cdot c_k
\]

where \( k \) is the tool type, for example, \( n_c \) and \( n_{cp} \) are the number of the full setups and partial setups for chopsaw respectively; \( c_c \) and \( c_{cp} \) are the corresponding costs of full setups and partial setups for chopsaw respectively.

To compute \( f_r \), we rely on the precision levels from Table S4 to assign a quantitative value for error-per-cut to each tool as shown in Table S3. Lower values are better (we only consider errors in measurement, not errors in fabrication, or stochastic errors).

The minimum measurement that can be made accurately using our tools is \( m = 1/16'' \) for all dimensions, and \( m = 1 \) for all angles.

<table>
<thead>
<tr>
<th>Tool</th>
<th>Full Fab. Time</th>
<th>Partial Fab. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chopsaw</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Bandsaw</td>
<td>4.0</td>
<td>N/A</td>
</tr>
<tr>
<td>Jigsaw</td>
<td>10.0</td>
<td>N/A</td>
</tr>
<tr>
<td>Tracksaw</td>
<td>5.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Drill</td>
<td>2.0</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table S2. Relative fabrication time costs for different tools.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Dimension</th>
<th>Relative Material Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2'' × 4''</td>
<td>24''</td>
<td>0.30</td>
</tr>
<tr>
<td>2'' × 4''</td>
<td>48''</td>
<td>0.55</td>
</tr>
<tr>
<td>2'' × 4''</td>
<td>96''</td>
<td>1.00</td>
</tr>
<tr>
<td>2'' × 2''</td>
<td>24''</td>
<td>0.30</td>
</tr>
<tr>
<td>2'' × 2''</td>
<td>48''</td>
<td>0.55</td>
</tr>
<tr>
<td>2'' × 2''</td>
<td>96''</td>
<td>1.00</td>
</tr>
<tr>
<td>4'' × 4''</td>
<td>24''</td>
<td>0.75</td>
</tr>
<tr>
<td>4'' × 4''</td>
<td>48''</td>
<td>1.375</td>
</tr>
<tr>
<td>4'' × 4''</td>
<td>96''</td>
<td>2.50</td>
</tr>
<tr>
<td>2'' × 8''</td>
<td>24''</td>
<td>0.75</td>
</tr>
<tr>
<td>2'' × 8''</td>
<td>48''</td>
<td>1.375</td>
</tr>
<tr>
<td>2'' × 8''</td>
<td>96''</td>
<td>2.50</td>
</tr>
<tr>
<td>1/2''</td>
<td>12'' × 20''</td>
<td>0.55</td>
</tr>
<tr>
<td>1/2''</td>
<td>24'' × 20''</td>
<td>1.00</td>
</tr>
<tr>
<td>1/2''</td>
<td>48'' × 96''</td>
<td>6.5</td>
</tr>
<tr>
<td>3/4''</td>
<td>12'' × 20''</td>
<td>0.7</td>
</tr>
<tr>
<td>3/4''</td>
<td>24'' × 20''</td>
<td>1.2</td>
</tr>
<tr>
<td>3/4''</td>
<td>48'' × 96''</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Table S1. Prices of stocks
Fig. S1. From left to right: 1) time versus #e-nodes for populating e-graphs; 2) optimization time versus #e-nodes; 3) relative hypervolume versus #e-nodes.

<table>
<thead>
<tr>
<th>Tool</th>
<th>Relative Error per cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Track saw</td>
<td>1.0</td>
</tr>
<tr>
<td>Chop saw</td>
<td>1.0</td>
</tr>
<tr>
<td>Band saw</td>
<td>2.5</td>
</tr>
<tr>
<td>Jigsaw</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Table S3. Relative error per cut for each tool. Lower values indicate less error per cut.

(Using tape measure and protractor). Thus, measurements that are a multiple of m have zero error. The error, $e_i$ for all other measurements, $m'$ is computed by the following formula: $e = m' \% m$. The mean precision is then measured as:

$$f_p = \frac{\sum_{i=1}^{n} (1 + e_i) \cdot p_i}{n}$$

where n is the number of cuts and $p_i$ is the precision of the tool used for the $i^{th}$ cut.

1.2 Results and Comparison to Expert Fabrication Plan

The additional results for the models in Figure 9 are shown in Figures S2 and S3. These results show some additional insights into the performance of the algorithm. The drafting table (9.B) example illustrates the trade-off between fabrication time and material cost. In solution B, by using smaller stocks, the parts on the block can be arranged in a way that reduces the setups, which reduces fabrication time. Solution A, on the other hand, uses fewer and larger boards and therefore, the material cost is reduced at the expense of fabrication time. For the birdhouse (9.D), HELM’s result reuses many of the same setups in solution B, saving time, while HELM improved precision by optimizing the order of cuts most influenced by kerf in solution A.

1.3 Optimization Time and Convergence

Cut planning is a combinatorial problem. The use of e-graphs and our pruning strategy make the problem tractable, but as discussed in the limitations section, this may not always return the optimal fabrication plan. To further evaluate our method, we vary the amount of pruning and plot graphs that show 1) performance versus the number of e-nodes (using the hypervolume indicator for multi-objective optimization [Auger et al. 2009]) and 2) computation time versus the number of e-nodes. As hypervolume varies due to different choices on reference points, we propose a normalization method to visualize the results - for each design $d_i$, we concatenate the scores of all Pareto-fronts, and use the concatenated matrix to compute its reference point $ref_i$. A maximal hypervolume among all Pareto-fronts calculated by $ref_i$ is $hv_i$, which can be used to normalize all other hypervolumes into a range of $[0, 1]$.

The computation time for computing the e-graph grows linearly with the number of e-nodes, while the performance quickly increases and then tapers off (see the right-most sub-figure in Figure S1). It is worth noting that the number of programs that an e-graph can represent is much greater than the number of e-nodes. We also reported time for populating e-graphs and optimization in Figure S1.

1.4 Concurrent Optimization of Multiple Models

To evaluate the convergence of our technique, we propose optimizing the fabrication process of multiple models. As an example, we consider the fabrication process of a dining set, consisting of a table (Figure 9.H) and six chairs (Figure 9.F). We argue that it is increasingly challenging for experts to optimize the fabrication instructions as the number of parts increases and, therefore, do not compare to expert results in this example. Instead, we compare running our algorithms for fabricating all parts in the dining set to the result of optimizing each object (chairs and table) individually. For the second case, we take the resulting Pareto-fronts and add them by considering all possible combinations of Pareto-optimal plans for each object.

While we expect that the first case (concurrent optimization) should generate better results since the search is done on a larger space, this may not be the case because of the need for additional pruning. The number of parts in the dining set is 108, which took 1.4 hours to populate e-graphs and additional 10.1 hours to optimize (#e-nodes is 359,489). The running time of optimization is high because the height of ASTs depends on the number of parts, and a tall AST needs more time to do cross-overs and mutations in our algorithm. Comparatively, we try all possible combinations of the points on Pareto-fronts from Section 2.2 (#e-nodes are 60,272 and 1,241) to construct a new Pareto-front for fabricating the whole dining set. Although concurrent optimization takes a substantial amount of time, since it gives results that fully dominate the ones from added
9.A: Adirondack chair

9.B: Drafting table

9.D: Birdhouse

9.F: Dining room chair

Fig. S2. Additional optimization results.
Fig. S3. Additional optimization results (continued).
Pareto-front exploration, it still yields significant benefits because relative to actual fabrication time, the optimization time is negligible. A smaller number of dining set (57 parts, Figure S4: bottom) was also tested using the described approach, and a similar result is obtained.

As an additional experiment, we also took the points from the separately-optimized pieces of the dining set, concatenated them into a single fabrication plan, and then further optimized their cut ordering to reduce fabrication time. This alternate strategy actually uncovered the lowest cost solutions with respect to fabrication time, demonstrating that the concurrent optimization strategy is not always able to explore the full Pareto frontier for large search spaces. We are eager to further explore how different optimizations scale in future work. We speculate that the difference between the concurrent strategy (red dots in Figure S4) and this strategy (blue dots in Figure S4) is that pruning is done by optimizing packing which reduces material costs, but makes it more challenging to find solutions that trade-off more material cost for fabrication time once the search space grows too large.

1.5 Implementation Details
To make our paper self-contained, we will elaborate implementation details from several perspectives in this section.

This paragraph gives more details of “Packing Pieces onto Stock”. Given a library of stock lumbers, we first group them by their dimensions. The parts are assigned to these groups using the method of Section 5.2. One part might be assigned to different groups, and we only choose the group with the maximum space utilization for which reduces material costs, but makes it more challenging to find solutions that trade-off more material cost for fabrication time once the search space grows too large.

Such a packing process traverses all of the parts in a specific sequence, which we call a full Traversal. At the same time, we get a set of Arrangements. If these Arrangements contain all parts of the current group, we repeat the packing process to pack the parts of each Arrangement into smaller stocks of the lumber group. With such a recursive packing process, e-graphs can be constructed by organizing all of the obtained Arrangements as stock e-nodes.

The key step of the packing process is to select the next part and its orientation to place into the current stock lumber. There are many possible choices for selecting the next part and its orientation. Our algorithm packs every un-placed part in each of its suitable orientations, then sorts all of the valid Arrangements following a radix sorting manner, which first sorts them by the number of aligned edges and then sort them by their bounding volume and distance to stock boundaries for those Arrangements with the same number of aligned edges. With different choices of the next part and its orientation, different Arrangements can be obtained. So the whole packing process is indeed a tree search process. Suppose that the number of input parts is \( N \), and each part has \( M \) valid orientations to assign to the current stock lumber, which results in \( N! \times M^N \) search space. It is time-consuming to enumerate all of these possible packing results with such a large search space. We use the number of Traversal \( T \) as our termination criterion. We use the same \( T \) as 50 in our experiments of Figure S2 and S3. The number of e-classes will increase with a greater \( T \). As explained in Section 5.2, we use a heuristic to prune some of the results according to the number of aligned edges. For each e-class, we keep the top \( n \) stock e-nodes (\( n = 10 \) is adopted in our experiments). For each stock e-nodes, we only compile it down for maximal \( P \) different orders of cuts, and we found \( P = 25 \) could give satisfactory results.

Our system adopts a multi-objective genetic algorithm, which could be easily parallelized. In our implementation, we create multiple threads to run the NSGA-II based optimization. We set the parameters of our genetic algorithm as \( p_c = 0.95 \), \( p_m = 0.1 \). Each thread maintains a population which has 120 active individuals. Our genetic algorithm has two termination criteria: 1) no performance gain within \( L \) iterations, or 2) exceed \( K \) iterations, whichever is first satisfied. In our experiments, we set \( L = 800 \) and \( K = 16,000 \). Moreover, a fast convergence sometimes requires proper strategies of initializing populations. In practice, carpenters would try to use a minimal set of tools to fabricate a design. Borrowing this idea, we use a portion of threads to to initialize population randomly, but try to use a specific process as many as possible, while the other threads completely randomize initialization similar to what a standard genetic algorithm does.

2 LANGUAGE DETAILS AND EXAMPLES
This section expands on Section 4 by defining each operation and parameter of the languages shown in Figure 4 in the paper, describes process characterization and the mapping from HL-HELM to LL-HELM.

2.1 High-Level HELM
Figure 4 (left) shows the grammar for HL-HELM programs, that are composed of a sequence of assignments that bind identifiers to the result of high-level fabrication operations: Make_Stock, Make_Hole, and Make_Cut. Our language is inspired by standard feature-based CAD scripting languages [FeatureScript 2019], where the features map to fabrication operations, such as getting stock and making cuts. As in CAD languages, we also include a special statement for defining 2D sketches. CAD systems create 3D geometry by applying features (e.g., extrusion or drafting) to 2D sketches. Make_Hole and Make_Cut use sketches to specify the path an operation should follow, e.g., a polyline to specify a bandsaw cut. Sketches are defined by a set of geometric primitives (points, line segments, circles, splines) and constraints.

A sketch is defined on the face on which an operation occurs. This information is provided by a query. A query is always made on a specific body. Every operation (Make_Hole, Make_Cut) takes an id for a body and all operations implicitly refers to that body. Some examples of queries are shown in Figure 4 (left).
Fig. S4. From left to right: dining set example including six chairs and one table; Two corresponding pareto-fronts, red dots correspond to the results of concurrent optimization, and green dots correspond to the results of pareto-fronts exploration. Note that the showing results on the right-hand side are the ones with minimal fabrication time.

Make_Stock
Make_Hole
Make_Cut
Lumber
Sheet
Chopsaw
Jigsaw
Bandsaw
Tracksaw
Drill

Fig. S5. (a) Process characterization diagram for saws. (b) Surjective mapping from HL-HELM to LL-HELM in our compiler.

2.2 Low-Level HELM

It supports fabrication operations like Chop saw, Jigsaw, Bandsaw, and Drill (hand drill). Features in HL-HELM like Make_Cut are mapped to concrete LL-HELM operations like Chop saw and Jigsaw. Similarly, Make_Hole is mapped to Drill, and Make_Stock is mapped to Lumber or Sheet, depending on the dimensions. Lumber and sheet are referenced using a catalog_id that uniquely identifies the part from a library of available material.

Some of these fabrication operations require a setup that configures the tool for performing the tasks. For example, the setup for chopsaw is specified by two angles: the miter angle, the bevel angle, and the offset from the stop block; and the setup for a drill is specified by the diameter of the hole. The diameter determines the choice of the drill bit. The bandsaw and jigsaw in Figure 4 do not require a setup. The cutting path for these two tools is specified by a list of reference points (ref_pt), where a ref_pt is defined by the intersection of two lines. These lines are specified by an offset from an edge on the part. Drill also requires a ref_pt that specifies the point where the hole should be made.

Additionally, LL-HELM supports a Stack operation that places two parts together before performing an operation when both parts use the same setup. This helps reduce the number of times the setup needs to be changed which is useful because changing setup can be time-consuming and error-prone. The language also provides an Unstack operation that separates the stacked parts after an operation has been performed. A LL-HELM program is then, a sequence of setups and assignments, followed by a Return statement that returns the resulting parts obtained from the fabrication operations.

2.3 Process Characterization

Table S4 describes the process characterization for every operation in our pipeline. The first four rows summarize the saw operations. Tracksaw and chopsaw are the most precise of all the cutting operations we support, followed by bandsaw and jigsaw. x and y are the maximum lengths the tool can support the respective dimension. z is the maximum height of the part that can be fit under the tool. x’ is the maximum distance between the leftmost end of the tool’s platform and the part. In the case of jigsaws, the maximum x and y dimensions are ∞ since the tool does not in any way constrain the part along x or y. Due to the same reason, x’ is also not constrained. The values for the other tools are in the table. The value of z is 1 inch for jigsaw and tracksaw. For chopsaw, it is 4 inches and for bandsaw it is 6 inches. Theta represents the miter angle and phi

Note that some bandsaws may allow setup where the table can be rotated by an angle.
Table S4. Process characterization table for all tools in LL-HELM.

<table>
<thead>
<tr>
<th>Tool</th>
<th>Precision</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>x'</th>
<th>Theta</th>
<th>Phi</th>
<th>R</th>
<th>Partial</th>
<th>Curve</th>
<th>Internal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jigsaw</td>
<td>low</td>
<td>(0, ∞)</td>
<td>(0, ∞)</td>
<td>(0, 1')</td>
<td>(0, ∞)</td>
<td>(0, 180)</td>
<td>90</td>
<td>(1', ∞)</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>Chop saw</td>
<td>high</td>
<td>(0.96 '')</td>
<td>(0, 6 '')</td>
<td>(0, 4 '')</td>
<td>(0, 36 '')</td>
<td>(-50, 60)</td>
<td>(45, 135)</td>
<td>-</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>Band saw</td>
<td>medium</td>
<td>(0.26 '')</td>
<td>(0, 24 '')</td>
<td>(0, 6 '')</td>
<td>(0, 13 '')</td>
<td>(0, 180)</td>
<td>90</td>
<td>(1', ∞)</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>Track saw</td>
<td>high</td>
<td>(0.96 '')</td>
<td>(0, 48 '')</td>
<td>(0, 1')</td>
<td>(0, 36 '')</td>
<td>90</td>
<td>(45, 135)</td>
<td>-</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>Drill</td>
<td>high</td>
<td>(0, ∞)</td>
<td>(0, ∞)</td>
<td>(0, ∞)</td>
<td>(0, ∞)</td>
<td>-</td>
<td>-</td>
<td>drill-bit diameter</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

represents the bevel angle. R is the minimum curvature of a path that the tool can follow. For chopsaw and tracksaw curved cuts are not possible, but for bandsaw and jigsaw the values are shown in the table. Figure S5(a) illustrates these parameters. Both jigsaw and bandsaw support partial cuts and curves whereas chopsaw and tracksaw do not. "Internal" indicates whether the tool can be used to make an internal cut on a part. Only jigsaws can be used to perform such cuts.

2.4 Mapping from HL-HELM to LL-HELM

There is an explicit surjective mapping from every feature in HL-HELM to an operation in LL-HELM. This mapping is shown in Figure S5(b). There are three types of features in our current implementation of HL-HELM which is easily extensible as Section 4 explains. In the figure, green denotes stock allocation, red denotes cuts, and yellow denotes holes.

2.5 LL-HELM Interpretation

LL-HELM instructions use UIDs to represent geometric information such as edges and faces, which are not difficult to interpret by carpenters. We build a LL-HELM visualization UI to depict step-by-step instructions that carpenters will follow. The left side of our UI shows optimized LL-HELM codes, and the right side visualizes a specific line of codes that needs to be performed. Users can easily explore the whole programs by clicking “previous step” and “next step” buttons.

2.6 HL- and LL-HELM of Designs

We show the high-level HELM for all of the models in Figure 9 and the resulting low-level HELM programs in Listings 1-20, which are at the end of the supplemental material. The principle of selecting a representative LL-HELM is choosing the one with the lowest cost on fabrication time from pareto-fronts.

2.7 User study

To evaluate the expressiveness of our system, we asked the three experts who created the physical models shown in the teaser, to fill out a survey on their experience with the tool and how it compares to conventional CAD systems. Here is a snippet from one of the experts:

"I saved significant time because the tool contained predefined stock lumber sizes. I could simply load a bunch of stock into the environment, cut it, and stitch it together in the assembly environment. Drawing the chair in CAD would be more difficult as I would have to specify additionally dimensions for each piece."

REFERENCES


Listing 1. Figure 9.A - Adirondack chair (HL-HELM)

```
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Box005</td>
<td>Make_Stock(457.2, 38.1, 88.9)</td>
<td></td>
</tr>
<tr>
<td>MyLine001</td>
<td>Line(0, 88.9, 88.9, 88.9)</td>
<td></td>
</tr>
<tr>
<td>Sketch007</td>
<td>Make_Sketch()</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cut016</td>
<td>Make_Cut(Box020, Sketch008, 45)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Listing 2. Figure 9.B - Drafting table (HL-HELM)

```
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Box009</td>
<td>Make_Stock(780.56, 88.9, 88.9)</td>
<td></td>
</tr>
<tr>
<td>MyLine007</td>
<td>Line(-780.56, 0, -630.283, 88.9)</td>
<td></td>
</tr>
<tr>
<td>Sketch009</td>
<td>Make_Sketch()</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cut015</td>
<td>Make_Cut(Box016, Sketch015, 45)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Listing 3. Figure 9.B - Drafting table (HL-HELM)

```
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Box010</td>
<td>Make_Stock(521.21, 38.1, 88.9)</td>
<td></td>
</tr>
<tr>
<td>MyLine008</td>
<td>Line(0, 88.9, 88.9, 88.9)</td>
<td></td>
</tr>
<tr>
<td>Sketch011</td>
<td>Make_Sketch()</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cut014</td>
<td>Make_Cut(Box015, Sketch015, 45)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```
Listing 6. Figure 9.C - Bookcase (LL-HELM)

```plaintext
Cut001 = Make_Cut(Box007, Sketch001, 0)
Geometry(MyLine001)
Constraint(Coincident(Start(MyLine001), End(MyLine001)), PointOnObject(End(MyLine001)), Query_Edge_By_Closest_Point(Cut001, 0, 0, 0), Angle(Start(MyLine001), End(MyLine001)), DistanceY(Start(Horizontal), MyCircle001, 139.7));
```

Listing 7. Figure 9.D - Birdhouse (HL-HELM)

```plaintext
Box007 = Make_Stock(426.72, 38.1, 184.15)
Setup_Chopsaw(60.0000, 0.0000, 0.0000)
(a19) = Chopsaw(lumber_2x4x24, face_9, edge_9)
Setup_Chopsaw(0.0000, 0.0000, 24.0000)
(a13) = Chopsaw(lumber_2x4x24, face_6, edge_6)
Setup_Chopsaw(-30.0000, 0.0000, 24.0000)
(a11, a12) = Chopsaw(cut, face_10, edge_10)
Setup_Chopsaw(0.0000, 0.0000, 24.2500)
(a10) = Chopsaw(cut, face_9, edge_9)
Setup_Chopsaw(0.0000, 0.0000, 23.5000)
(a9) = Chopsaw(cut, face_8, edge_8)
Setup_Chopsaw(0.0000, 0.0000, 24.0000)
(a8) = Chopsaw(cut, face_7, edge_7)
Setup_Chopsaw(0.0000, 0.0000, 24.0000)
(a7, a8) = Chopsaw(cut, face_6, edge_6)
Setup_Chopsaw(0.0000, 0.0000, 22.5000)
(a6) = Chopsaw(cut, face_5, edge_5)
Setup_Chopsaw(0.0000, 0.0000, 24.0000)
(a5, a6) = Chopsaw(cut, face_4, edge_4)
Setup_Chopsaw(0.0000, 0.0000, 24.0000)
(a4) = Chopsaw(cut, face_3, edge_3)
Setup_Chopsaw(0.0000, 0.0000, 24.0000)
(a3) = Chopsaw(cut, face_2, edge_2)
Setup_Chopsaw(0.0000, 0.0000, 24.0000)
(a2) = Chopsaw(cut, face_1, edge_1)
Setup_Chopsaw(0.0000, 0.0000, 24.0000)
(a1) = Chopsaw(cut, face_0, edge_0)
```

Listing 8. Figure 9.D - Birdhouse (LL-HELM)

```plaintext
Constraint(PointOnObject(Start(MyLine000), MyLine000)));
Constraint(PointOnObject(Start(MyLine001), MyLine001)));
Constraint(PointOnObject(Start(MyLine002), MyLine002)));
Constraint(PointOnObject(Start(MyLine003), MyLine003)));
Constraint(PointOnObject(Start(MyLine004), MyLine004)));
Constraint(PointOnObject(Start(MyLine005), MyLine005)));
Constraint(PointOnObject(Start(MyLine006), MyLine006)));
Constraint(PointOnObject(Start(MyLine007), MyLine007)));
Constraint(PointOnObject(Start(MyLine008), MyLine008)));
Constraint(PointOnObject(Start(MyLine009), MyLine009)));
Constraint(PointOnObject(Start(MyLine010), MyLine010)));
Constraint(PointOnObject(Start(MyLine011), MyLine011)));
Constraint(PointOnObject(Start(MyLine012), MyLine012)));
Constraint(PointOnObject(Start(MyLine013), MyLine013)));
Constraint(PointOnObject(Start(MyLine014), MyLine014)));
Constraint(PointOnObject(Start(MyLine015), MyLine015)));
Constraint(PointOnObject(Start(MyLine016), MyLine016)));
Constraint(PointOnObject(Start(MyLine017), MyLine017)));
Constraint(PointOnObject(Start(MyLine018), MyLine018)));
Constraint(PointOnObject(Start(MyLine019), MyLine019)));
Constraint(PointOnObject(Start(MyLine020), MyLine020)));
Constraint(PointOnObject(Start(MyLine021), MyLine021)));
Constraint(PointOnObject(Start(MyLine022), MyLine022)));
Constraint(PointOnObject(Start(MyLine023), MyLine023)));
Constraint(PointOnObject(Start(MyLine024), MyLine024)));
Constraint(PointOnObject(Start(MyLine025), MyLine025)));
Constraint(PointOnObject(Start(MyLine026), MyLine026)));
Constraint(PointOnObject(Start(MyLine027), MyLine027)));
Constraint(PointOnObject(Start(MyLine028), MyLine028)));
Constraint(PointOnObject(Start(MyLine029), MyLine029)));
Constraint(PointOnObject(Start(MyLine030), MyLine030)));
Constraint(PointOnObject(Start(MyLine031), MyLine031)));
Constraint(PointOnObject(Start(MyLine032), MyLine032)));
Constraint(PointOnObject(Start(MyLine033), MyLine033)));
Constraint(PointOnObject(Start(MyLine034), MyLine034)));
Constraint(PointOnObject(Start(MyLine035), MyLine035)));
Constraint(PointOnObject(Start(MyLine036), MyLine036)));
Constraint(PointOnObject(Start(MyLine037), MyLine037)));
Constraint(PointOnObject(Start(MyLine038), MyLine038)));
Constraint(PointOnObject(Start(MyLine039), MyLine039)));
Constraint(PointOnObject(Start(MyLine040), MyLine040)));
Constraint(PointOnObject(Start(MyLine041), MyLine041)));
Constraint(PointOnObject(Start(MyLine042), MyLine042)));
Constraint(PointOnObject(Start(MyLine043), MyLine043)));
Constraint(PointOnObject(Start(MyLine044), MyLine044)));
Constraint(PointOnObject(Start(MyLine045), MyLine045)));
Constraint(PointOnObject(Start(MyLine046), MyLine046)));
Constraint(PointOnObject(Start(MyLine047), MyLine047)));
Constraint(PointOnObject(Start(MyLine048), MyLine048)));
Constraint(PointOnObject(Start(MyLine049), MyLine049)));
Constraint(PointOnObject(Start(MyLine050), MyLine050)));
Constraint(PointOnObject(Start(MyLine051), MyLine051)));
Constraint(PointOnObject(Start(MyLine052), MyLine052)));
Constraint(PointOnObject(Start(MyLine053), MyLine053)));
Constraint(PointOnObject(Start(MyLine054), MyLine054)));
Constraint(PointOnObject(Start(MyLine055), MyLine055)));
Constraint(PointOnObject(Start(MyLine056), MyLine056)));
Constraint(PointOnObject(Start(MyLine057), MyLine057)));
Constraint(PointOnObject(Start(MyLine058), MyLine058)));
Constraint(PointOnObject(Start(MyLine059), MyLine059)));
Constraint(PointOnObject(Start(MyLine060), MyLine060)));
Constraint(PointOnObject(Start(MyLine061), MyLine061)));
Constraint(PointOnObject(Start(MyLine062), MyLine062)));`
Listing 12. Figure 9.F - Dining room chair (LL-HELM)

```
VisibleFaceByClosestPoint(Box018, 74.995, 0, 19.05), Start((Box018, 0, 0, 19.05));
```

Listing 13. Figure 9.G - Bench (HL-HELM)

```
VisibleFaceByClosestPoint(Box022, 74.995, 0, 19.05), Start((Box022, 0, 0, 19.05));
```
Listing 18. Figure 9.1 - Flower pot (LL-HELM)

1. Setup_Tracksaw(0.0000, 0.0000, 20.2750)
2. (a0, a1) = Tracksaw(Sheet_0.5x24x20, Face_0, edge_0)
3. Setup_Tracksaw(0.0000, 0.0000, 5.0000)
4. (a1, a2) = Tracksaw(a0, face_1, edge_1)
5. Setup_Tracksaw(0.0000, 0.0000, 6.1933)
6. (a5) = Tracksaw(a2, face_2, edge_2)
7. Setup_Tracksaw(0.0000, 0.0000, 13.0000)
8. (a6) = Tracksaw(a5, face_3, edge_3)
9. (a6, a7) = Tracksaw(a5, face_4, edge_4)
10. Setup_Tracksaw(0.0000, 0.0000, 12.0000)
11. (a7, a12) = Tracksaw(a6, face_5, edge_5)
12. Setup_Tracksaw(0.0000, 0.0000, 12.0000)
13. (a5, a15) = Tracksaw(a2, face_6, edge_6)
14. Setup_Tracksaw(0.0000, 0.0000, 0.0000)
15. (a15, a16) = Tracksaw(a2, face_7, edge_7)
16. Setup_Tracksaw(0.0000, 0.0000, 0.0000)
17. (a16, a17) = Tracksaw(lumber_2x4x24, face_10, edge_10)
18. (a13) = Chop Saw(a17, edge_13)
19. Return(a6, a7, a11, a14, a15, a17, a19)

Listing 19. Figure 9.1 - Z-table (HL-HELM)

1. Box001 = Make_Cut(BBox001, Sketch001, 0); Sketch001 = Make_Sketch(
2. Box002 = Make_Stock(792.48, 38.1, 88.9);
3. Box003 = Make_Stock(396.24, 0, 88.9), Angle(End(
4. MyLine002), End(Query_Edge_By_Closest_POINT(BBox003, 396.24, 0, 88.9)), Coincident(End(
5. MyLine002), End(Query_Edge_By_Closest_POINT(BBox003, 396.24, 0, 88.9), 102.667)));
6. Cut003 = Make_Cut(BBox003, Sketch001, 0);
7. Box004 = Make_Stock(319.243, 0, 0)), Start(
8. MyLine002), 60) ;
9. Sketch002 = Make_Sketch(
10. Cut004 = Make_Cut(Box004, Sketch002, 0);
11. Box005 = Make_Stock(914.4, 38.1, 88.9)
12. Sketch003 = Make_Sketch(
13. Box006 = Make_Stock(914.4, 38.1, 88.9), Angle(Start(
14. MyLine002), End(Query_Edge_By_Closest_POINT(Cut004, 344.087, 0, 88.9)), Angle(Start(
15. MyLine002), End(Query_Edge_By_Closest_POINT(Cut003, 319.243, 0, 0)), Start(
16. MyLine002), 120) ) ;
17. Sketch004 = Make_Sketch(
18. Cut005 = Make_Cut(Cut004, Sketch004, 0);
19. Box007 = Make_Stock(495.3, 38.1, 88.9)
20. Sketch005 = Make_Sketch(
21. Box008 = Make_Stock(495.3, 914.4, 12.7)
22. Sketch006 = Make_Sketch(
23. Box009 = Make_Stock(495.3, 914.4, 12.7), Angle(Start(
24. MyLine002), End(Query_Edge_By_Closest_POINT(Cut008, 266.7, 0, 88.9)), Angle(Start(
25. MyLine002), End(Query_Edge_By_Closest_POINT(Cut007, 266.7, 0, 88.9)), Angle(Start(
26. MyLine002), End(Query_Edge_By_Closest_POINT(Cut006, 266.7, 0, 88.9)), Angle(Start(
27. MyLine002), End(Query_Edge_By_Closest_POINT(Cut005, 266.7, 0, 0)))};

Listing 20. Figure 9.1 - Z-table (LL-HELM)

1. Setup_Tracksaw(0.0000, 0.0000, 19.5000)
2. (a0, a1) = Tracksaw(sheet_0.5x24x20, Face_0, edge_0)
3. Setup_Tracksaw(0.0000, 0.0000, 2.5000)
4. (a3, a4) = Tracksaw(a0, face_1, edge_1)
5. Setup_Tracksaw(0.0000, 0.0000, 4.0000)
6. (a5) = Tracksaw(a2, face_2, edge_2)
7. Setup_Tracksaw(0.0000, 0.0000, 7.5000)
8. (a8) = Tracksaw(a5, face_4, edge_4)
9. (a7, a12) = Tracksaw(a6, face_5, edge_5)
10. Setup_Tracksaw(0.0000, 0.0000, 12.0000)
11. (a15) = Chop Saw(a2, face_6, edge_6)
12. Setup_Tracksaw(0.0000, 0.0000, 0.0000)
13. (a15, a16) = Tracksaw(a2, face_7, edge_7)
14. Setup_Tracksaw(0.0000, 0.0000, 0.0000)
15. (a16, a17) = Chop Saw(lumber_2x4x24, face_10, edge_10)
16. (a13) = Chop Saw(a17, edge_13)
17. Return(a6, a7, a11, a14, a15, a17, a19)