The Space of All Stereo Images

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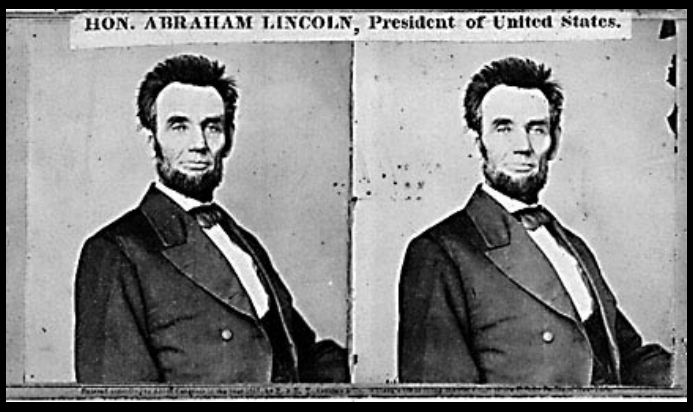
Stereo Imaging

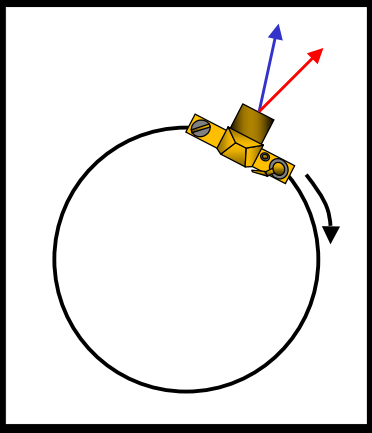


Key property: horizontal parallax

- Enables stereoscopic viewing and stereo algorithms
- Gives rise to epipolar geometry

What other image varieties have this property?

Stereo Panoramas



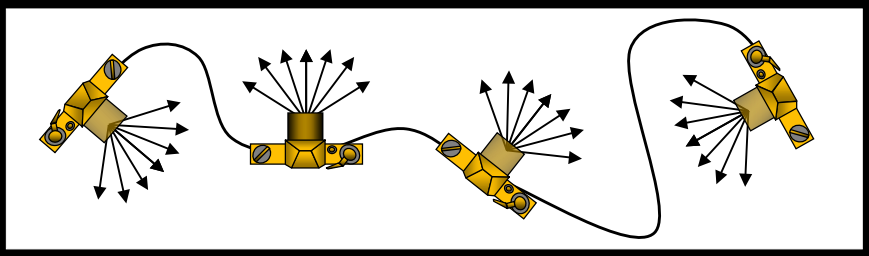
left

right

- [Ishiguro, Yamamoto, Tsuji, 92]
- [Peleg and Ben-Ezra, 99]
- [Shum, Kalai, Seitz, 99]
- [Nayar and Karmarkar, 00]

I

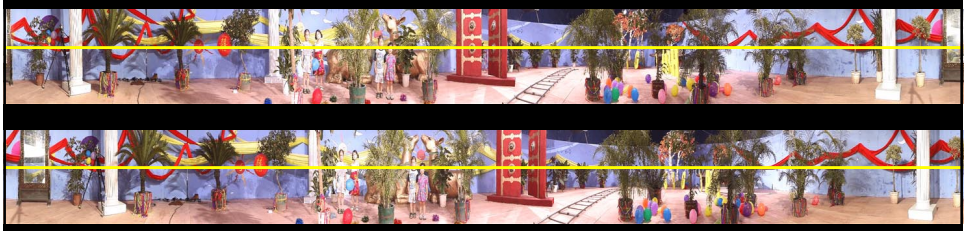
Problem Statement

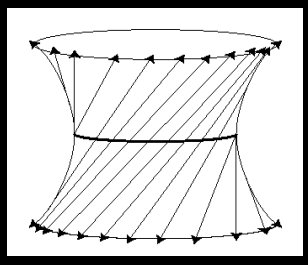


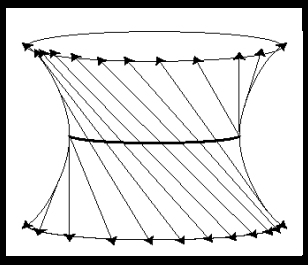
Suppose you could capture *any* set of rays

- Which rays result in a stereo pair?
  - satisfy horizontal parallax constraint
- How to define epipolar geometry?
- How to express projection, triangulation?

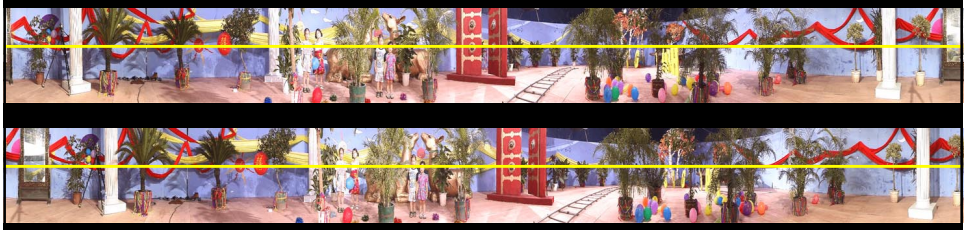
Geometry of Stereo Panoramas

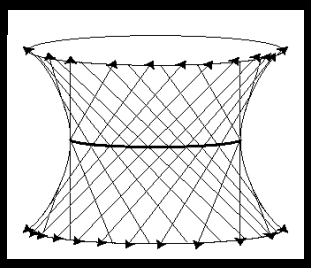




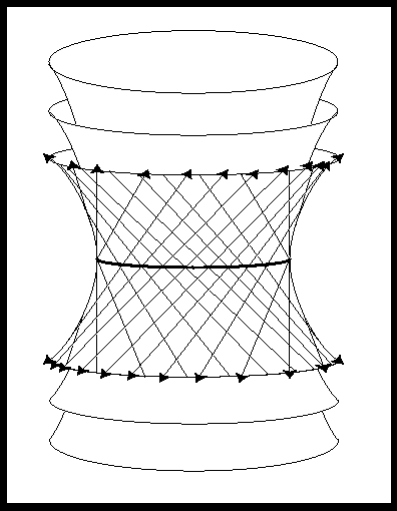


Geometry of Stereo Panoramas





Epipolar Geometry

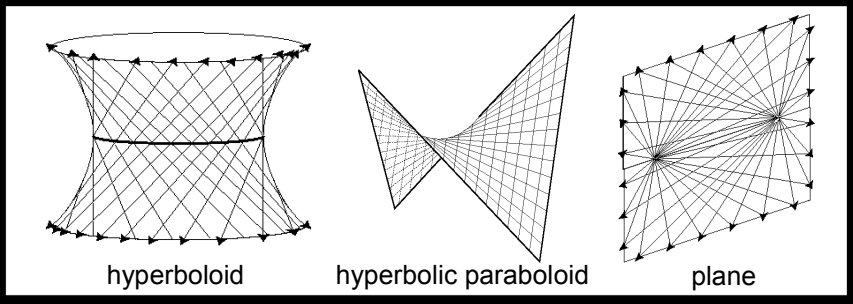


Stereo Classification Theorem

Two images form a stereo pair iff

- Each row lies on a doubly ruled surface
- Corresponding rows are the same surface

There are only 3 doubly-ruled surfaces



⇒ Extension of classical result [Hilbert]

Generating Stereo Views

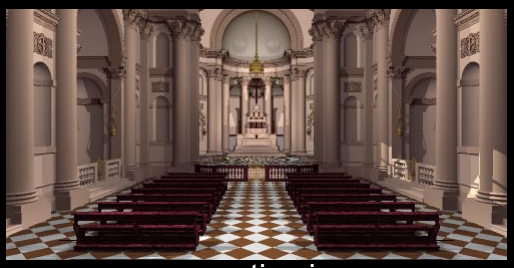
Recipe book for stereo views

- *All* stereo pairs are generated by moving a camera along a conic curve
  - line, ellipse, parabola, hyperbola

image representation	generator	epipolar surface
perspective	point	pencil of planes
stereo panorama [IYT 92], [PB 99], [SKS 99]	circle	half-hyperboloids
omnivergent, 360 × 360 [SKS 99], [NK 00]	circle	hyperboloids
spherical omnivergent [SKS 99]	sphere	pencil of planes
pushbroom stereo	line	pencil of planes
stereo cyclograph	ellipse	half-hyperboloids
parabolic panorama	parabola	hyperbolic paraboloids

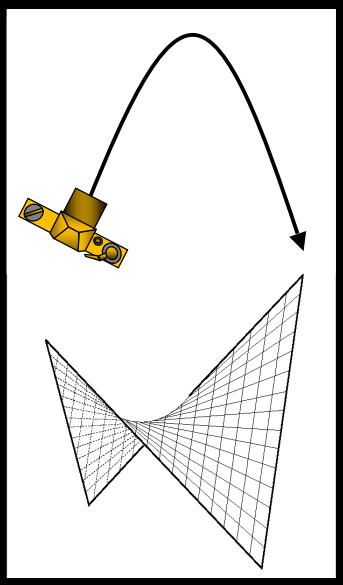
Pushbroom Stereo

Parabolic Panorama

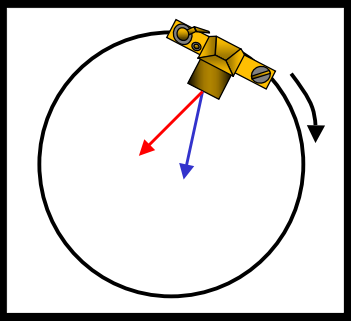


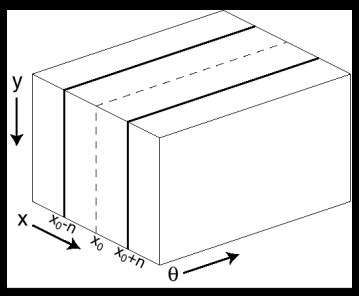
perspective image

parabolic panorama



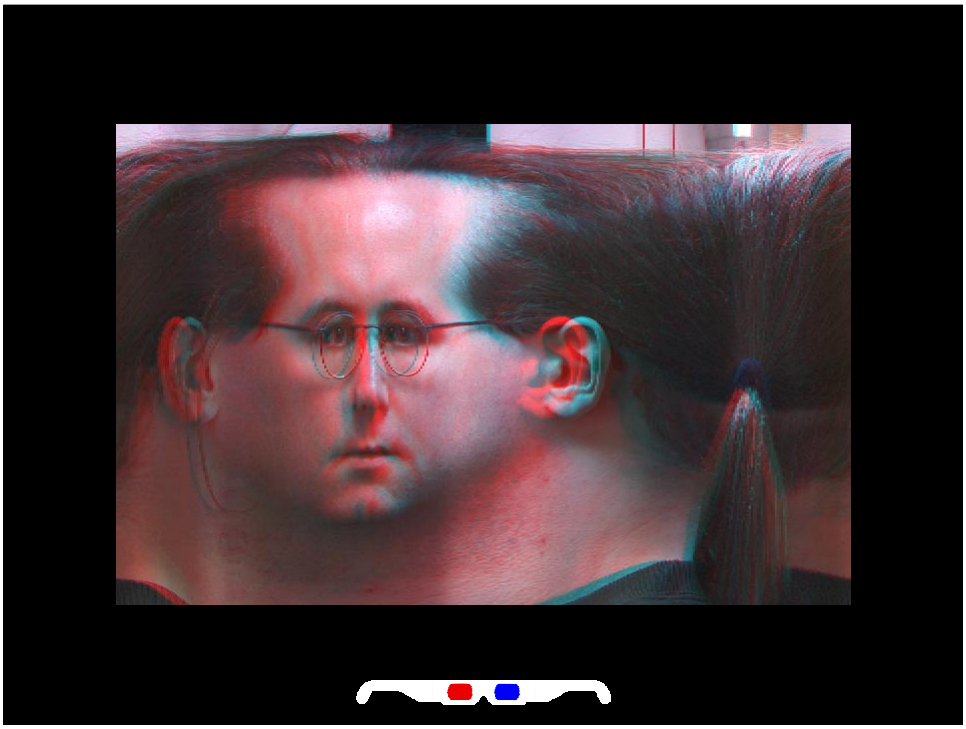
Stereo Cyclographs

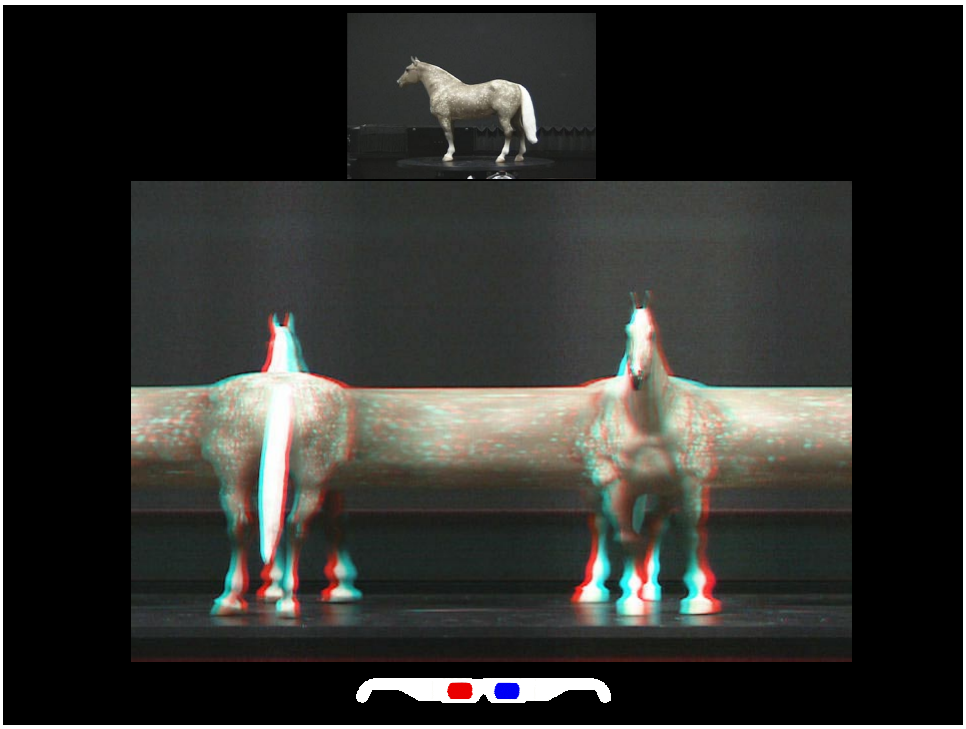




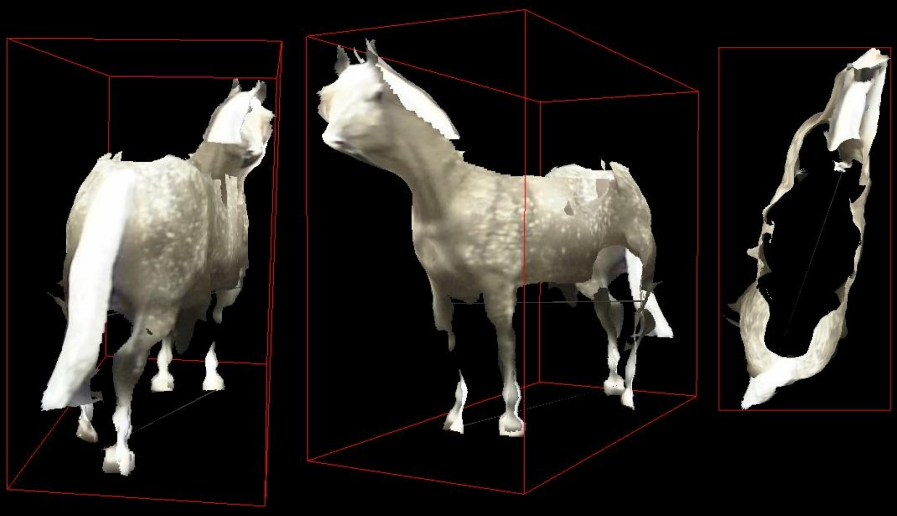
input

cyclographs





Stereo Cyclograph Reconstruction



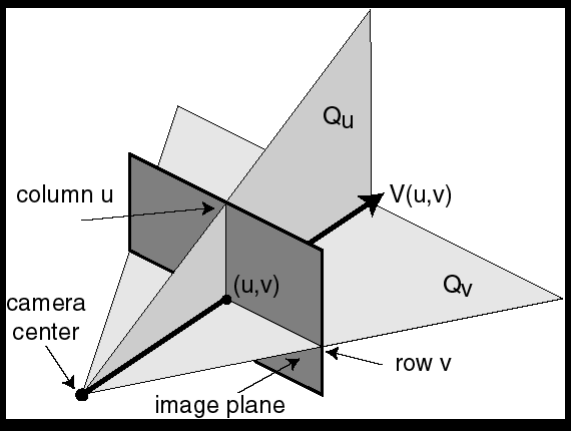
Computed from two cyclograph images

- Using unmodified stereo matcher [Zitnick & Kanade]

Mathematics for Stereo Views

Defined by two families of quadric surfaces

- Rays in row u lie on quadric Q_u
- Rays in column v lie on quadric Q_v



Mathematics for Stereo Views

Projection

- Solve for u, v, given X

$$\begin{aligned} \mathbf{X}^T\mathbf{Q}_v\mathbf{X} &= 0 \\ \mathbf{X}^T\mathbf{Q}_u\mathbf{X} &= 0 \end{aligned}$$

Triangulation

- Solve for X, given u_1, u_2, v

$$\begin{aligned} \mathbf{X}^T\mathbf{Q}_v\mathbf{X} &= 0 \\ \mathbf{X}^T\mathbf{Q}_{u_1}\mathbf{X} &= 0 \\ \mathbf{X}^T\mathbf{Q}_{u_2}\mathbf{X} &= 0 \end{aligned}$$

Unified treatment for all stereo varieties

- Reduces to standard eqns for the perspective case

Conclusions

Main results

- Stereo Classification Theorem
- Recipe book for generating stereo pairs
- Mathematics of stereo imaging

Future work

- Exploring interesting new varieties
- Multiperspective camera design
- Multiperspective image analysis

Thanks to Jiwon Kim